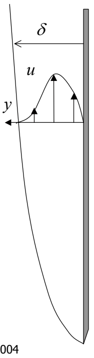
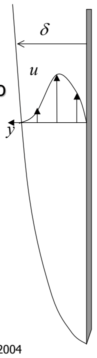
Free Convection

- In the previous discussions, a freestream velocity set up the conditions for convective heat transfer.
- Due to friction with the surface, the flow must be maintained by a fan or pump – thus it is called forced convection.
- An alternate situation occurs when a flow moves naturally due to buoyancy forces due to the heat transfer itself.
- This so called "free" convection is illustrated in the figure to the side for a heated wall.



Free Convection [2]

- Buoyancy is the result of difference is density between materials.
- In this case, the difference in density is due to difference is temperature.
- In the figure, the air next to the plate is heated, its density decreases, and the resulting buoyancy forces the air to rise.
- However, note that only the flow inside the thermal boundary layer moves – the velocity is zero both at the wall and far away from it.
- Also note that for simplicity, I have rotated the axis system as well as the flow.



Governing Equations

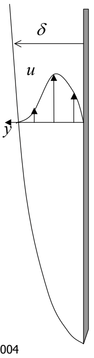
- The governing equations for this case become:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - g + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial x}\right)^2$$

- The only difference from before is the inclusion of gravitational acceleration in the 2nd, momentum, equation.



Governing Equations [2]

- The pressure gradient in the x (vertical) direction is due to the hydrostatic forces in the freestream. Thus: $\frac{\partial p}{\partial x} = -\rho_\infty g$
- The two "forcing" terms are then:

$$\frac{1}{\rho}\frac{\partial p}{\partial x} + g = -\frac{\rho_\infty g}{\rho} + g = \frac{(\rho - \rho_\infty)g}{\rho}$$

- The density difference can be related to the temperature difference with the volume coefficient of expansion:

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = \frac{(V - V_\infty)}{V(T - T_\infty)} = \frac{(\rho_\infty - \rho)}{\rho(T - T_\infty)}$$

Governing Equations [3]

- The net forcing due to the temperature differences is then: $\frac{1}{\rho}\frac{\partial p}{\partial x} + g = -g\beta(T - T_\infty)$
- The momentum equation is then:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g\beta(T - T_\infty) + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

- To switch to non-dimension terms, a reference velocity is needed – but the freestream is at rest!
- Instead, a "viscous" velocity is defined by:

$$V_{ref} = \frac{\mu_\infty}{\rho_\infty L} = \frac{\upsilon_\infty}{L}$$

Non-dimensional Coefficients

- The non-dimensional momentum equation is then:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = \left[\frac{g\beta(T_s - T_\infty)L}{V_{ref}^2}\right]\left(\bar{T} - \bar{T}_\infty\right) + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

- The term in the square bracket is a new coefficient called the <u>Grashof number</u>:

$$\boxed{Gr_L = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon_\infty^2} = \frac{g\beta\rho_\infty^2(T_s - T_\infty)L^3}{\mu_\infty^2}}$$

- And the equation becomes:

$$\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = Gr_L\left(\bar{T} - \bar{T}_\infty\right) + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2}$$

Non-dimensional Coefficients [2]

- The Grashof number is to free convection what the Reynolds number is to forced convection.
- Practically, the Grashof number is the ratio of forcing (buoyancy) forces to restraining (viscous) forces.
- As with Re, the magnitude Grashof number will indicated the relative impact of viscosity and the nature of the flow – laminar or turbulent.
- A corollary to the Peclet number also exists – the Raleigh number, defined by:

$$Ra = Gr_L \Pr$$

Analytic/Empirical Solutions

- Analytic and empirical solutions will thus be a function of Gr as well as the Pr and will often take the form:

$$\overline{Nu}_L = f(Gr_L, \Pr) \quad \text{or} \quad f(Ra)$$

- For example, the book discusses the approximate solution for laminar flow on a flat plate. The result is:

$$Nu_x = 0.508 \Pr^{1/2} (0.952 + \Pr)^{-1/4} Gr_x^{1/4}$$

- or

$$\overline{Nu}_L = 0.677 \Pr^{1/2} (0.952 + \Pr)^{-1/4} Gr_L^{1/4}$$

Analytic/Empirical Solutions[2]

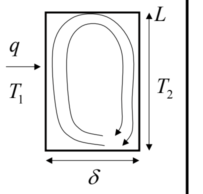
- However, the most common form is simply:

$$\overline{Nu} = C(Gr \Pr)^{\mathrm{m}}$$

- Values for the two constants, C and m, for different geometries and Grashoff number ranges is given in Table 7-1.
- These solutions include vertical plates or cylinders, horizontal cylinders and even horizontal plates.
- The text includes numerous other, more complex relations for special cases or with higher accuracy.
- Some particularly simple solutions for h which depend only upon ∆T are given in Table 7-2.

Enclosed Spaces

- A particular interesting case occurs when heating surfaces in an enclosed space.
- In this case, the solution must account for the replacement of the heated, rising fluid (or cooled, descending fluid) by re-circulation cells:



- At very low Grashof numbers, there might be no flow at all – only conduction.
- This case is similar to thermal inversions which sometimes occur in valley cities.

Enclosed Spaces [2]

- At higher Grashof numbers, a viscous dominated flow may exist, similar to creeping flow seen at low Re.
- With increasing Gr, the flow topology passes through the laminar to turbulent flow along the walls.
- The heat flux from one side of the enclosure to the other can be expressed by an equivalent conduction coefficient:

$$\frac{k_e}{k} = C\left(Gr_\delta \,\mathrm{Pr}\right)^n \left(\frac{L}{\delta}\right)^m$$

- Values of the coefficients can be found in Table 7-3

Combined Convection

- Finally, there are also a large number of situations when the flow cannot be characterized as either simply forced or free, but as a combination of the two.
- This is a common situation in electronics where a low velocity fan might be used to augment the natural free convection of components .
- As would be expected, empirical relations for these cases depend upon both Gr and Re.
- Unfortunately, there is very limited data for these situations and the book only provides results for tube flow – not a common situation in electronics.