

## Accelerated Flight

- Thus far, all the performance parameters we have considered have been for an aircraft in unaccelerated flight - so called static performance.
- Now lets begin to consider the performance of an aircraft experiencing accelerations - either along or perpendicular to the flight path.
- For the first case - accelerations along the flight path - we will consider the two most extreme situations: takeoff and landing.
- Later, for the second case - accelerations normal to the flight path - we will consider turning flight.

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## Takeoff Performance

- An aircraft under acceleration must obey Newton's second law:

$$F = ma \quad \text{or} \quad a = F/m$$

- Further, the differential change in position of the aircraft while accelerating is given by calculus:

$$dS = Vdt \quad \text{but} \quad dV = a dt \quad \text{so} \quad dS = \frac{V}{a} dV$$

- So the distance needed to reach a given speed is:

$$S_G = \int_0^{S_G} dS = \int_0^{V_G} \frac{V}{a} dV$$

- In these relations,  $S_G$  is the distance required to reach a target ground velocity,  $V_G$ , starting from rest.

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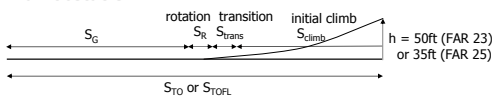
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## Takeoff Performance (continued)

- In reality, flight regulations do not specify takeoff distance by  $S_G$ , but by the distance needed to clear an obstacle:



- Also, for safety, the lift-off airspeed,  $V_{LOF}$  is taken as a fraction (1.1 or 1.2) above stall speed  $V_{stall}$ .
- Finally, if there are winds,  $V_{wind}$  the ground speed differs from the target lift-off speed:

$$V_G = V_{LOF} \pm V_{WIND} \quad \begin{array}{l} + \text{ for tail wind} \\ - \text{ for head wind} \end{array}$$

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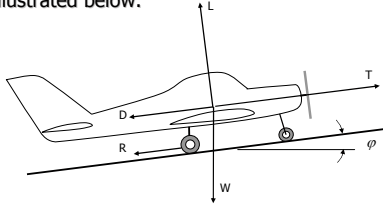
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### Takeoff Performance (continued)

- Now let's concentrate on our particular situation as illustrated below:



- In addition to the familiar forces of L, W, T, and D, we also have a ground resistance force, R.

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### Takeoff Performance (continued)

- This resistance force is due to the rolling friction between the tires and the ground.
- Assume this force is proportional to the normal force of contact:  

$$R \approx \mu_r (W - L)$$
- The friction coefficient used here,  $\mu_r$ , will also depend upon the type of runway surface. Typical values are:  
 $\mu_r = 0.02$  for smooth, hard runway (asphalt or concrete)  
 $\mu_r = 0.1$  for grass runway (unmowed and uncompressed)

- With this new force, a summary of the forces in the flight path direction gives:

$$F = T - D - W \sin(\phi) - \mu_r (W - L)$$

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### Takeoff Performance (continued)

- One special note concerning drag applies to both takeoff and landing.
  - Due to the interaction of the ground, the wing tip vortices are weakened.
  - The result is both an enhanced lift, and a decreased induced drag - this is called the **ground effect**.
  - To account for this drag decrease we will use:

$$D = q_\infty S \left( C_{D,0} + \phi \frac{C_L^2}{\pi e A R} \right)$$

where  $\phi$  depends upon the height of the wing above the ground, h, and the wing span, b:

$$\phi = \frac{(16h/b)^2}{1 + (16h/b)^2} \quad 0 \leq \phi \leq 1$$

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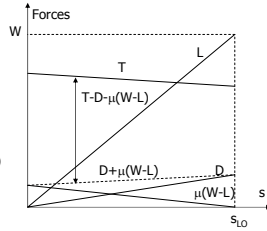
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### Takeoff Performance (continued)

- If the airplane forces are plotted schematically for a jet aircraft (fighter), we would see something like:
- Note that all our forces vary as the aircraft gains speed.
- The net axial force also varies, but not as much.
- Thus, it is reasonable to replace this net force with an average value.




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### Takeoff Performance (continued)

- Thus, assuming that thrust is nearly constant, and angles are small the average axial force becomes:
- $$\bar{F} = T - [D + W\phi + \mu_r(W-L)]_{avg} = const$$
- To determine an average value for the other forces,  $D$  and  $R$ , it is suggested that the value at 0.707 the lift off velocity is used:

$$[D + \mu_r(W-L)]_{avg} = [D + \mu_r(W-L)]_{0.7V_{LOF}}$$

- And, in turn, the lift off velocity is typically 1.2 times the stall speed at takeoff (or 1.1 for military a/c):

$$V_{LOF} = 1.2V_{stall} = 1.2 \sqrt{\frac{2W}{\rho_x SC_{L,max,TO}}}$$

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### Takeoff Performance (continued)

- Putting all these assumptions together finally gives us the take off ground roll distance:

$$S_G = \frac{V_G^2}{2a} = \frac{mV_G^2}{2\bar{F}} = \frac{V_G^2(W/g)}{2[T - [D + \mu_r(W-L)]_{avg}]}$$

or

$$S_G = \frac{1.44W^2}{g\rho_x SC_{L,max} [T - [D + \mu_r(W-L)]_{avg}]}$$

- If the total force cannot be assumed constant, we must numerically integrate:

$$S_G = \int_0^{V_G} \frac{V}{a} dV = \int_0^{V_G} \frac{V(W/g)}{T - [D + W\phi + \mu_r(W-L)]} dV$$

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## Takeoff Performance (continued)

- To summarize our results for take off performance:

$$S_G = \frac{1.44W^2}{g\rho_\infty S C_{L,max} \left\{ T - [D + \mu_r(W-L)]_{avg} \right\}}$$

- Takeoff distance increases with the square of aircraft weight!
- If thrust variations with altitude are included,  $T \propto \rho_\infty$  then takeoff distance increases inversely with the square of density,  $S \propto 1/\rho_\infty^2$ !
- Takeoff distance decrease with higher wing area,  $S$ , or higher takeoff  $C_{L,max}$ .
- Takeoff distance decreases with either higher thrust or reduced ground resistance.

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## Takeoff Performance (continued)

- Finally, the book notes that the take off distance can be minimized by maximizing the acceleration.
- The acceleration can in turn be maximized by configuring the aircraft for an "optimum"  $C_L$ .
- Without going through the math, the book shows that the optimum  $C_L$  can be found to be:

$$C_{L,opt} = \frac{\mu_r}{2K} = \frac{1}{2} \mu_r \pi e A R$$

- This  $C_L$  would be obtained by flap settings and/or wing incidence during the take-off roll.
- Use this  $C_L$  in calculating the average  $L$  and  $D$ .

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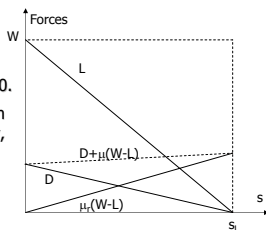
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## Landing Performance

- Calculations for landing performance are very similar to those for takeoff performance.
- The primary differences are:

- Most airplanes land with the engines at idle -  $T \sim 0$ .
- The aircraft starts with an initial touchdown velocity,  $V_{TD}$  and decelerates to rest.
- The resistance forces, drag and ground friction are intentionally large.




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### Landing Performance (continued)

- The balance of forces now gives:

$$F = T - D - \mu_r(W - L) = m \frac{dV}{dt}$$

- And, as with takeoff, we will assume a constant average axial force can be defined by:

$$\bar{F} = T - [D + \mu_r(W - L)]_{avg} = T - [D + \mu_r(W - L)]_{0.7V_{TD}}$$

- The distance relation is the same except for a negative sign to account for starting with velocity  $V_T$  and decelerating to rest:

$$S_{G_{landing}} = \int_0^{S_{G_{landing}}} dS = \int_{V_{TD}}^0 \frac{V}{a} dV$$

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### Landing Performance (continued)

- Putting all these together gives the relation:

$$S_{G_{landing}} = \frac{V_T^2 m}{2\bar{F}} = \frac{V_{TD}^2 (W/g)}{2[D + \mu_r(W - L)]_{avg} - T}$$

- Most aircraft approach the runway at 1.3 times the stall speed for safety reasons.
- However, touch-down, after flaring, usually occurs at 1.15 times the stall speed (all aircraft), such that:

$$V_T = 1.15V_{stall} = 1.15 \sqrt{\frac{2W}{\rho_\infty SC_{L,max,L}}}$$

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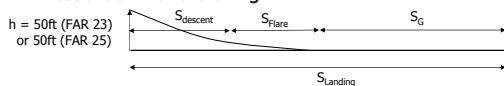
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### Landing Performance (continued)

- The landing maneuver usually involves an approach to the runway, a gliding decent, flare, and then touchdown and braking:



- Most aircraft approach the runway at 1.3 times the stall speed for safety reasons.
- However, touch-down, after flaring, usually occurs at 1.15 times the stall speed (all aircraft), such that:

$$V_{TD} = 1.15V_{stall} = 1.15 \sqrt{\frac{2W}{\rho_\infty SC_{L,max,L}}}$$

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## Landing Performance (continued)

- As before, if we assume a constant, average force:

$$S_{G_{landing}} = \frac{(1.15W)^2}{g\rho_x S C_{L,max,L} \left\{ [D + \mu_r(W-L)]_{avg} - T \right\}}$$

- To increase the resisting forces and thus shorten landing distances, a number of force enhancing methods are used:
  - First, brakes are normally applied. This has the effect of increasing the rolling friction. For paved, dry runways, a value of  $\mu_r=0.4$  is typical.
  - Also the profile drag associated with landing flaps is intentionally high - thus increasing  $C_{d,0}$ .

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## Landing Performance (continued)

- Spoilers are used on many aircraft. These upper surface, split flap like devices act to abruptly decrease the wing lift once on the ground which helps to improve the rolling friction. Spoilers also increase the profile drag.
- Drag chutes use to be common on high performance fighters. And of course landing arresting systems such as on carrier ships can be used.
- Finally, many airplanes can produce reversed thrust either through a mechanical system on jet engines or reversed prop pitch on piston/turboprop engines. To include reversed thrust, simply add in this term with the other resisting forces:

$$T_R = -T$$

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## Landing Performance (continued)

- Alternately, you may use the approach shown in the book which assumes  $C_D$  and  $C_L$  are constants during landing ground roll.

$$S_{G_{landing}} = \frac{W}{g\rho_x S (C_D - \mu_r C_L)} \ln \left[ 1 + \frac{\rho_x S V_{TD}^2 (C_D - \mu_r C_L)}{2(\mu_r W - T)} \right]$$

- As with take-off, if neither assumption is accurate for the aircraft being analyzed, you must use numerical integration of the original equation:

$$S_{G_{landing}} = \int_{V_{TD}}^0 \frac{V}{a} dV = \int_{V_{TD}}^0 \frac{V}{T - [D + \mu_r(W-L)]} dV$$

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