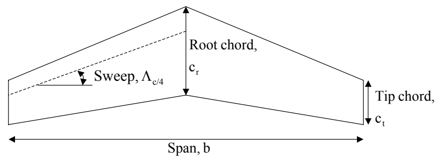
Finite wings

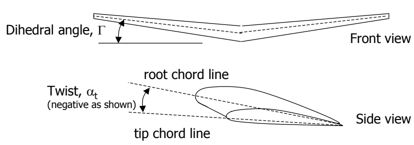
- We have a new set of terms associated with wings:
  - The diagram show some **planform** definitions.



  - Other definitions derived from these parameters are:
    - Mean chord, $\overline{c} = (c_r + c_t)/2$
    - Wing area, $S = b\,\overline{c}$
    - Taper ratio, $\lambda = c_t/c_r$

Finite wings

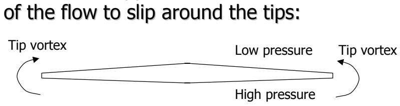
- Some other geometric parameters for wings are:



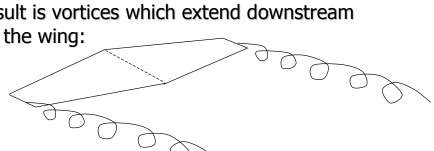
- One last parameter of interest to us is called the Aspect Ratio, $AR = b^2/S$.
  - The aspect ratio is basically a measure of the "squareness" of a wing (AR=1 is a square, but AR>>1 is better!).
  - AR appears many times in the analysis of wing performance.

Finite wings and Vortices

- When a wing is producing lift, the difference in pressure between upper and lower surfaces causes some of the flow to slip around the tips:

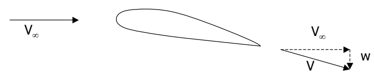


- The result is vortices which extend downstream behind the wing:

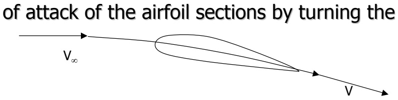


Finite wings and Vortices (continued)

- An important effect of these vortices is the production of **downwash**, w, between the wing tips



- This "pushing down" of the airflow is what give the equal and opposite force pushing up, the lift.
- The downwash also acts to reduce the effective angle of attack of the airfoil sections by turning the airflow.

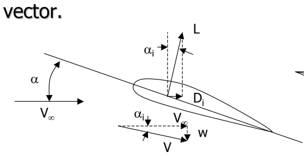


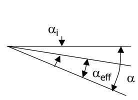
Finite wings and Vortices (continued)

- Another important impact of downwash is a new form of drag, called induced drag, C_{Di}.
- Induced drag has been explained three ways:
  - A by-product of the undesirable changes in upper surface pressure due to the lower surface flow slipping around at the wing tips.
  - The energy wasted in producing the wing tip vortices which have very high rotational kinetic energies.
  - The tilting of the lift vector due to the local angle of attack changes produced by the wing tip vortices.
- Which is correct? Well, all three are just a different prospective on the same phenomena.

Induced Drag

- Mathematically, induced drag can be calculated by using the third explanation - the tilting of the lift vector.





- The difference between the geometric angle-of-attack, α, and the vortex induced angle-of-attack, α_i, is called the effective angle-of-attack, α_{eff}.

Induced Drag (continued)

- The induced drag is just the component of the tilted lift vector in the freestream flow direction. Thus

$$D_i = L \sin \alpha_i$$

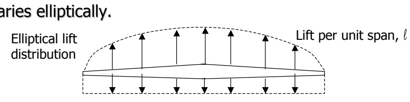
- If we assume small angles, then

$$D_i = L\alpha_i \quad \text{or} \quad C_{D,i} = C_L \alpha_i$$

  - where α_i is in radians!
- So, to calculate induced drag, all we need is to know α_i. Unfortunately, that is much easier said then done.

Induced Drag (continued)

- The mathematics and theory necessary to calculate α_i are beyond the scope of this course, but we can state a few basic ideas:
  - In general, α_i and the downwash, w, need not be constant along the span because the lift per unit span, ℓ, varies.
  - Wing design parameters which effect ℓ are taper, wing twist, and differences in airfoil shape between root and tip.
  - The lowest possible induced drag occurs when α_i and w **are** constant across the span. This in turn happens when ℓ varies elliptically.



Induced Drag (continued)

- For the case of an elliptical lift distribution, the induced angle-of-attack everywhere is given by:

$$\alpha_i = \frac{C_L}{\pi AR}$$

- Thus, the induced drag for this case is:

$$\boxed{C_{D,i} = C_L \alpha_i = \frac{C_L^2}{\pi AR}}$$

- For non-elliptical distribution, we introduce a correction called the span efficiency factor,e:

$$\boxed{C_{D,i} = \frac{C_L^2}{\pi e AR}}$$

Note that e ≤ 1.0

Induced Drag (continued)

- From these last relations, it is obvious that the induced drag is directly related to the amount of lift being produced. Thus, it is often called the **drag due to lift**.
- The total drag on a wing is then the sum of the drag due to friction (profile drag), plus the drag due to lift.

$$C_D = C_{D,p} + C_{D,i} = C_{D,p} + \frac{C_L^2}{\pi e AR} \qquad (+C_{D,w})$$

If transonic/ supersonic

- Keep in mind that the profile drag also varies with angle-of-attack and thus lift!

Lift curve slope, $dC_L/d\alpha$

- Due to α_i, the airfoils composing the wing have a lift which depends upon the α_{eff} rather than α. Thus:

$$C_L = a_o(\alpha_{eff} - \alpha_0) \quad \text{and} \quad \frac{dC_L}{d\alpha_{eff}} = a_0$$

- However, we do not know α_{eff} before hand, and the difference, $\alpha_{eff} = (\alpha - \alpha_i)$, varies with the amount of lift being produced.
- Anyway, what we really want is how lift varies with the geometric angle of attack or

$$C_L = a_o(\alpha - \alpha_i - \alpha_0)$$

Lift curve slope, $dC_L/d\alpha$ (continue)

- For non-elliptical lift distributions we can introduce another efficiency factor, e_1, for the induced angle-of-attack such that:

$$\alpha_i = \frac{C_L}{\pi e_1 AR}$$

  - Note that in general e and e_1 can be different. However in practice, $e \approx e_1$.
- With this we have:

$$C_L = a_0\left(\alpha - \frac{C_L}{\pi e_1 AR} - \alpha_0\right)$$

- Solving this equation for C_L yields the desired relation between C_L and α:

$$C_L = \frac{a_0(\alpha - \alpha_0)}{1 + a_0/\pi e_1 AR}$$

Lift curve slope, $dC_L/d\alpha$ (continued)

- Or, differentiating this yields the 3-D lift curve slope, a.

$$\frac{dC_L}{d\alpha} = a = \frac{a_0}{1 + a_0/\pi e_1 AR}$$

Note: both a and a_o have units of 1/rad. To get 1/deg, multiply both by $180/\pi$=57.3!

- From this we see that the wing lift curve slope is always less than that for the airfoils it is made from!