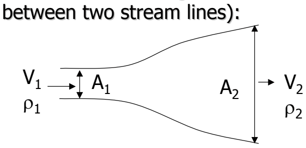
Conservation Equations

- Just as in physics, we will describe the motion of fluids through the requirement of conserving mass, momentum and energy.
- To simplify the math, we will restrict ourselves to case of **steady flow** - thus the total mass, momentum and energy do not change at any point over time!!
- The final equations we derive will also not always look like what you expect they should. But **remember where they came from and what they represent** if you want to understand aerodynamics.

Mass Conservation

- To conserve mass, **what flows in must equal what flows out**.
- So, consider the flow through a stream tube (the region between two stream lines):



mass flow in = mass flow out

Mass Conservation (continued)

- The mass flow rate in is: $\dot{m}_1 = \rho_1 V_1 A_1$
- The mass flow rate out is: $\dot{m}_2 = \rho_2 V_2 A_2$
- For mass conservation: $\dot{m}_1 = \dot{m}_2$ or

$$\rho_1 V_1 A_1 = \rho_2 V_2 A_2 \qquad \text{Continuity}$$

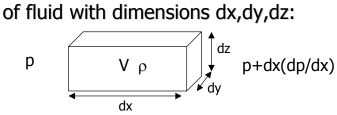
- Why call it “continuity”?
  - Well, if you didn’t conserve mass, you would end up with voids in the flow. Thus, to have a continuous flow you must have mass conservation.

Incompressible Flow

- Liquids are incompressible fluids. But **gasses are compressible fluids**, I.e. density changes under the influence of pressure.
- However, in many fluid flow situations, the changes in density are so small, accurate results can be obtained assuming ρ = constant.
- These situations are called **incompressible flow** and are typical of low speed flight (V < 300 ft/sec) and no heat addition.
- For incompressible flow, the continuity equation is even simpler: $V_1A_1 = V_2A_2$

Momentum Conservation

- To insure momentum conservation, consider a small volume of fluid with dimensions dx,dy,dz:



- Forces produce changes in momentum by: F = ma
- First, consider the net force on the volume

$$\Sigma F = pdydz - \left(p + dx\frac{dp}{dx} \right) dydz = -dxdydz\frac{dp}{dx}$$

Momentum Conservation (continued)

- Next, the momentum change can be rewritten by:

$$ma = \rho(dxdydz)\frac{dV}{dt} =$$

$$\rho(dxdydz)\frac{dV}{dx}\frac{dx}{dt} = \rho(dxdydz)V\frac{dV}{dx}$$

- Putting these results together gives:

$$-dxdydz\frac{dp}{dx} = \rho(dxdydz)V\frac{dV}{dx}$$

- Or simply

Euler's Equation: $\boxed{dp = -\rho V dV}$

Bernoulli's Equation

- Euler's equation if valid for all steady flows.
- A special form of this equation can be derived for the special case of incompressible flow, I.e. ρ= constant.
- In this case, Euler's equation can be directly integrated,

$$\int dp = -\int \rho V dV = -\rho \int V dV$$

- to get **Bernoulli's Equation**:

$$\boxed{p + \tfrac{1}{2}\rho V^2 = \text{constant}}$$

Bernoulli's Equation (continued)

- Some special notes about using Bernoulli's eqn:
  1) The pressure, p, is often the called the **static** pressure
  2) The term, 1/2 ρ V^2, is often called the **dynamic** pressure and given the symbol q.
     - The dynamic pressure is not a physical property. It is a measure of the pressure potential of a moving fluid.
  3) The constant on the right hand side is called either the **stagnation** pressure, p_o, or the **total** pressure, p_t.

  Thus, an alternate way to write Bernoulli's eqn is:

$$p + q = p_o$$